Solutions to Problem 1.

a. $\Pr\{Y_2 = 5\} = \dfrac{e^{-2(2)}(2(2))^5}{5!} \approx 0.16$

b. $\Pr\{Y_4 - Y_3 = 1\} = \Pr\{Y_1 = 1\} = \dfrac{e^{-2}2^1}{1!} \approx 0.271$

c. $\Pr\{Y_6 - Y_3 = 4 \mid Y_3 = 2\} = \Pr\{Y_3 = 4\} = \dfrac{e^{-2(3)(2(3))^4}}{4!} \approx 0.134$

d. $\Pr\{Y_5 = 4 \mid Y_4 = 2\} = \Pr\{Y_5 - Y_4 = 2 \mid Y_4 = 2\} = \Pr\{Y_1 = 2\} = \dfrac{e^{-2}2^2}{2!} \approx 0.271$

Solutions to Problem 2. Let $\lambda = 2$. Let t be measured in hours from 6 a.m.

a.
$$\begin{aligned}\Pr\{Y_4 = 9 \mid Y_2 = 6\} &= \Pr\{Y_4 - Y_2 = 3 \mid Y_2 = 6\}\\ &= \Pr\{Y_2 = 3\}\\ &= \frac{e^{-2(2)}(2(2))^3}{3!} \approx 0.195\end{aligned}$$

b. The expected time between successive arrivals is $E[G_n] = 1/2$ hour. The probability that the time between successive arrivals will be more than 1 hour is

$$\Pr\{G_n > 1\} = 1 - \Pr\{G_n \le 1\} = 1 - F_{G_n}(1) = 1 - (1 - e^{-2(1)}) = e^{-2} \approx 0.135$$

c. The expected time until the first patient arrives is $E[G_1] = 1/2$ hour. The probability that the first patient arrives in 15 minutes or less is

$$\Pr\{G_1 \le 1/4\} = F_{G_1}(1/4) = 1 - e^{-2(1/4)} \approx 0.393$$

d. $\Pr\{T_{13} \le 7\} = F_{T_{13}}(7) = 1 - \displaystyle\sum_{k=0}^{12} \frac{e^{-2(7)}(2(7))^k}{k!} \approx 0.641$

Solutions to Problem 3. Let $\lambda = 1/50$ defect per m^2. Let t be measured in m^2 of metal.

$$\begin{aligned}\Pr\{Y_{200} \ge 7\} &= 1 - \Pr\{Y_{200} \le 6\}\\ &= 1 - \sum_{k=0}^{6} \frac{e^{-(1/50)(200)}(200/50)^k}{k!}\\ &\approx 0.111\end{aligned}$$

Solutions to Problem 4.

a.
$$\begin{aligned}\Pr\{Y_4 > 30 \mid Y_2 = 10\} &= \Pr\{Y_4 - Y_2 > 20 \mid Y_2 = 10\}\\ &= \Pr\{Y_4 - Y_2 > 20\}\\ &= \Pr\{Y_2 > 20\}\\ &= 1 - \Pr\{Y_2 \le 20\}\\ &= 1 - \sum_{k=0}^{20} \frac{e^{-8(2)}(8(2))^k}{k!} \approx 0.1318\end{aligned}$$

b. $\Pr\{T_{50} \le 6\} = F_{T_{50}}(6) = 1 - \sum_{k=0}^{49} \frac{e^{-8(6)}(8(6))^k}{k!} \approx 0.405$

Note. You should get the same answer if you computed $\Pr\{Y_6 \ge 50\}$ instead.

c.
$$\begin{aligned}
\Pr\{T_{100} \le 12 \mid Y_6 = 40\} &= \Pr\{Y_{12} \ge 100 \mid Y_6 = 40\} \\
&= \Pr\{Y_{12} - Y_6 \ge 60 \mid Y_6 = 40\} \\
&= \Pr\{Y_{12} - Y_6 \ge 60\} \\
&= \Pr\{Y_6 \ge 60\} \\
&= 1 - \sum_{k=0}^{59} \frac{e^{-8(6)}(8(6))^k}{k!} \approx 0.0523
\end{aligned}$$

d. $E[T_4] = \frac{4}{8} = \frac{1}{2}$

Solutions to Problem 5. The rate of errors after the nth proofreading is

$$\lambda = \frac{1}{2^n} \text{ errors per 1000 words}$$

So, the probability of no errors after the nth proofreading is

$$\Pr\{Y_{200} = 0\} = \frac{e^{-200\frac{1}{2^n}}(200\frac{1}{2^n})^0}{0!} = e^{-\frac{200}{2^n}}$$

We want to find the smallest n such that $\Pr\{Y_{200} = 0\} \ge 0.98$:

$$\begin{aligned}
e^{-\frac{200}{2^n}} &\ge 0.98 \\
-\frac{200}{2^n} &\ge \ln(0.98) \\
2^n &\ge -\frac{200}{\ln(0.98)} \approx 9900 \\
\Rightarrow n &\ge 14 \quad \text{(by trial-and-error)}
\end{aligned}$$

Solutions to Problem 6.

a. Probably a good approximation. Independent increments is likely satisfied, because there is a large number of potential customers who act independently. Stationary increments is likely satisfied, as long as we restrict our attention to periods of the day when the arrival rate is roughly constant.

b. Not a good approximation. Independent increments is likely violated, since most arrivals occur during a brief period just prior to the start of the game, and only a few before or after this period.

c. Not a good approximation. Independent increments is likely violated if patients are scheduled, and therefore their arrivals are anticipated.

d. Not a good approximation. Stationary increments is likely violated, because the rate of finding bugs will decrease over time.

e. Probably a good approximation. Independent increments is likely satisfied because fires happen (largely) independently, and there are a large number of potential arrivals (buildings on fire). Stationary increments is likely satisfied, as long as we restrict our attention to periods of the day when the fire incident rate is roughly constant (e.g., daytime vs. nighttime).